TALENT DEVELOPMENT CENTRE
INDIAN INSTITUTE OF SCIENCE, KUDAPURA
Challakere, Chitradurga District, Karnataka-577536

MATHS ASSIGNMENT: 01

1. Show that sinA cosA tanA + cosA sinA cotA = 1.

2. Show that $\sec^2 A + \text{cosec}^2 A = \sec^2 A \text{ cosec}^2 A$.

3. If tanA + cotA = 2, show that secA cosec A = 2.

4(i) If $\cos\theta = \frac{4}{5}$ and $0 < \theta < 90^0$, find the value of $\frac{3\cos\theta + 2\,\text{cosec}\,\theta}{4\sin\theta - 2\cot\theta}$

(ii) If $\tan\theta = \frac{p}{q}$ and θ is acute, find the value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$

5. Show that $\frac{cos\theta - 2cos^3\theta}{2sin^3\theta - sin\theta} = cot\theta$

6. Show that $\sec^6\theta - \tan^6\theta = 1 + 3\tan^2\theta \sec^2\theta$.

7. Show that $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13$.

8. Show that $\cos^2 A(3 - 4\cos^2 A)^2 + \sin^2 A(3 - 4\sin^2 A)^2 = 1$.

9. If x = ar sinθ cosϕ, y = br sinθ sinϕ, z = crcosθ, show that $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = r^2$

10. If 3sinθ + 4cosθ = 5 and θ is acute, show that $\cot\theta = \frac{4}{3}$

11. If $\sin x + \sin^2 x = 1$, find the value of $\cos^{12} x + 2\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$.

12. Show that $\frac{cosA}{1 - tanA} - \frac{sin^2 A}{cosA - sinA} = cosA + sinA$

13. Show that $\frac{tan\theta}{1 - cot\theta} + \frac{cot\theta}{1 - tan\theta} = sec\theta cosec\theta + 1$

14. If $\sec\theta = x + \frac{1}{4x}$, prove that $sec\theta + tan\theta = 2x \text{ or } \frac{1}{2x}$.

15. If sinθ and cosθ are the roots of the equation $px^2 - qx + r = 0$, p, q ,r satisfy the relation

A. $p^2 - q^2 + 2pr = 0$ B. $p^2 + q^2 + 2pr = 0$ C. $p^2 - q^2 - 2pr = 0$ D. $p^2 + q^2 + 2pr = 0$

16. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

17. If $7\sin^2\theta + 3\cos^2\theta = 4$, find $sec\theta + cosec\theta$.

18. If θ is an acute angle, show that cosθ + sinθ > 1.

19.If $\sin x + \sin^2 x = 1$, then the value of $\cos^2 x + \cos^4 x$ =?.

20. If π = 180^0, and $A = \frac{\pi}{6}$, prove that $\frac{(1 - \sin A)(1 + \sin A)}{(1 - \cos A)(1 + \cos A)} = 3$

21. If A = 60^0 B = 30^0, then prove that sinA cosB − cosA sinB = sin(A − B).

22. Prove that $\tan 15^0 \tan 20^0 \tan 75^0 \tan 70^0 = 1$.

23. If sec4A = cosec (A − 20^0) and 4A is acute angle, find A.

24.What is the maximum value of $\frac{2}{cosec\theta}$? Justify your answer.

25.Evaluate : $\frac{tan^2 60+4cos^2 45+3sec^2 30+5cos^2 90}{cosec30+sec60-sec^2}$

26. Using the t-ratios of standard angles, Evaluate $\sin 75^0$,if $\sin(A + B) = \sin A \cos B + \cos A \sin B$

27. Prove that (i) $\frac{secA+tanA+1}{secA-tanA+1} = secA + tanA$ (ii) $\frac{cosA-sinA+1}{cosA+sinA-1} = cosecA + cotA$

28. If $x = asin\theta + bcos\theta \; and \; y = acos\theta - bsin\theta$. Prove that $x^2 + y^2 = a^2 + b^2$.

29.Evaluate: (i) $\frac{sec70}{cosec20} + \frac{sin59}{cos31}$ (ii) $\sqrt{\left(\frac{4cos43}{sin47}\right)^2 + \left(\frac{3cos37cosec53}{tan5\ tan25\ tan45\ tan65\ tan85}\right)^2}$

30.If $sec2A = cosec(A - 42)$ where 2A is an acute angle, find angle A.

31.Evaluate : $cosec(65 + \theta) - sec(25 - \theta) - \tan(55 - \theta) + \cot(35 + \theta)$

32 Simplify :

$\sin^2\theta \ \sin^2 \propto + \sin^2\theta \cos^2 \propto + \cos^2\theta \sin^2 \propto + \cos^2\theta \cos^2 \propto + 3cosec^4\theta - 3\cot^4\theta - 3\cot^2\theta$

33. Find the value of θ if (i) $\frac{cos\theta}{1-\sin\theta} + \frac{cos\theta}{1+sin\theta} = 4;\ \theta \leq 90.$ (ii) $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3\ ;\ \theta \leq 90.$

34. In ΔPQR, if PQ : QR :PR = 5 : 12 : 13, then evaluate cos P cos R – sin P sinR.

35. If A ,B, C are interior angles of a $\Delta\ ABC$. Prove that $\tan\left(\frac{A+C}{2}\right) = \cot\frac{B}{2}$.

36. If cosec θ+ cot θ= q, then find sin $\theta \; and \; tan\theta$.

37. The angles of elevation of the top of a cliff as seen from the top and bottom of a building are 45^0 and 60^0 respectively. If the height of the building is 24m, find the height of the cliff.

38. From the top of a 16m high building, the angle of elevation of the top of the building is 60^0, and the angle of depression of the foot of the hill is 30^0. find the height of the hill.

39. From a point 60m above a lake, the angle of elevation of top of a light house is 60^0. The angle of depression of its reflection in the lake is 30°. Find the height of the light house from the surface of the lake.

40. An aeroplane is flying at a certain height from the plane. The pilot observes angles of depression of two consecutive kilometre stones on opposite sides of the aeroplane as α and β. Prove that the height of the aeroplane is $\frac{tan\alpha\ tan\beta}{tan\alpha + tan\beta}$.

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