TALENT DEVELOPMENT CENTRE
INDIAN INSTITUTE OF SCIENCE, KUDAPURA
Challakere, Chitradurga District, Karnataka-577536
MATHS ASSIGNMENT: 03

1. The denominator of a fraction exceeds twice the numerator by 1. If 11 is added to the numerator, the fraction becomes 2. Find the original fraction.

2. The sum of the digits of a two digit number is 9. If the digits are reversed, the number is increased by 27. Find the number.

3. The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each of them saves 1000, find their respective incomes.

4. A two digit number is such that when a decimal point is placed between its digits the resulting number is 1/4 times the sum of its digits. Find the number

5. The ages of A and B are in the ratio 9:4. After 7 years, their ages will be in the ratio 5:3. Find their present ages.

6. Find the remainder when $3x^3 + 5x^2 - 2x + 4$ is divided by $(x - 1)$. If (x - 2) is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$, find a.

7. If $(x - 2)$ is the HCF of $x^2 + x - a$ and $x^2 + bx - 2$, find the roots of $x^2 + bx - a$.

8 If α and β are the roots of the equation $x^2 - x + 1 = 0$,then evaluate $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Also, find the equation whose roots are $\frac{\alpha^2}{\beta}$ & $\frac{\beta^2}{\alpha}$.

9. If $(x - 2)$ is a factor of $x^2 + ax + b - 9$ and $(x + 1)$ is a factor of $ax^2 + 5x - 2b$, find $a^2 + b^2$.

10. Find the remainder when $5x^5 + 4x^4 + 3x^3 + 2x^2 + x + 1$ is divided by $x^3 - 6x^2 + 11x - 6$.

11. If the equation $x^2 - 6x + c$ has equal roots, find the roots of $x^2 - cx + 18 = 0$.

12. If x+1, 2x and 2x+1 are the sides of a right-angled triangle, find x. Also determine the area of this triangle.

13. Draw the graphs of $x + y - 4 = 0$ and $2x + 2y - 5 = 0$. Do they intersect? What is the conclusion you can draw?

14.(a) Draw the graph of $4y = x^2$. Locate $\sqrt{24}$ on the x-axis using this graph.
(b). Draw the graphs of $y = x^2$ and $y = 3x - 2$. Do they intersect? What do you infer?
(c). Draw the graphs of $y = 2x^2$ and $x - y - 5 = 0$. Do they intersect? What do you infer?

15. In solving a quadratic equation $x^2 - ax + b = 0$, a boy copies a wrongly and obtained the roots as 6 and 2. Another boy copies b wrongly and obtained the roots as 5 and 2. Find the correct roots of the equation.

16. If $x = 2^{1/3} + 4^{1/3}$, find the value of $x^3 - 6x + 8$.

17. If a is root of $x^3 - 3x^2 + 5x + 11 = 0$ and b is root of $x^3 - 3x^2 + 5x - 17 = 0$, find a + b.

18. Find all solutions of $x^3 + 7x^2 - 18x + 10 = 0$.

19. If $3^x = 7^y = 63^z$, prove that $xy = 2yz + zx$.

20. Is there a perfect square the sum of whose digits is 2015?

21. A boy goes to sleep sometime between 9 PM and 10 PM. He wakes up some time between 5 AM and 6 AM next morning. He finds that the minute hand and hour hand overlap at both times. How long did he sleep?

22. How many four-digit numbers are there that consist of different digits, end in 5 and are divisible by each of their digits?

23. Find the sum of first 19 terms of an AP whose tenth term is 150.

24. In an AP, the first term is 2, the last term is 29 and the sum of the remaining terms is 124. Find the common difference.

25. How many terms of the sequence 2, 6, 18 ... are to be added to get the sum equal to 728?

26. The sum of first four terms of an AP is equal to half the sum of its next four terms. If the first term of the AP is 5, then find the AP .

27. Find the sum of an AP whose first term is -10, the last term is 20 and the sum of the 3rd, 4th and 6th term is 0.

28. Suppose a; b; c are in AP and a + b + c = 21. If a; b - 1; c + 1 is a GP, find a; b; c.

29. How many integers less than 1000 are there which are neither divisible by 5 nor by 7?

30. Determine all right-angled triangles whose sides are in AP.

31.a)If in an AP, m times the m-th term is equal to n times the n-th term, then prove that (m+n)th term of the AP is zero

b)If in an AP, sum of 'm' term is equal to sum of 'n' term, then prove that the sum of (m+n) terms of the AP is zero.

32 If the pth term of an AP is $\frac{1}{q}$ and its qth term is $\frac{1}{p}$, show that the sum of its first pq terms is $\frac{1}{2}(pq+1)$

33. Find the number of terms of the AP 17, 15, 13, ... so that their sum is 72. Explain the double answer

34. If the pth, qth and rth terms of an AP is a, b, c respectively, then show that a(q –r) + b(r –p) + c(p –q) = 0.

35. The sum of n terms of an AP is $\left[\frac{5n^2}{2}+\frac{3n}{2}\right]$. Find its 20th term.

36. If the first, second and fifth terms of an AP are in GP, prove that the sum of the first five terms of an AP is 25 times the first term.

37. For each $n \in N, let\ S_n = 1 - 2 + 3 - 4 + \cdots + (-1)^{n-1}n$. Determine$S_{2014} + S_{2015}$.

38. Find the sum of all 5 digit numbers that can be formed using the digits 1,2,3,4,5, each digit exactly once.

39.(a)Show that each of the following equation has real roots, and solve each by using the quadratic formula: $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2)$

(b). For what values of 'm' will $2mx^2 - 2(1+2m)x + (3+2m) = 0$ have real and distinct roots?

(c) Evaluate: $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6\ldots\ldots\ldots}}}}$

40. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

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