



**TALENT DEVELOPMENT CENTRE  
INDIAN INSTITUTE OF SCIENCE, KUDAPURA  
Challakere, Chitradurga District, Karnataka-577536**

**Problems on Mathematical Induction**

1. There are  $n$  identical cars on a circular track. Pooling together, they have just enough for one car to complete one round the track. Show that There is a car which can complete one round by collecting petrol from other cars on its way around.
2. Prove for any natural number  $N$ , the inequality

$$\sqrt{2\sqrt{3\sqrt{4\cdots\sqrt{(N-1)\sqrt{N}}}}} < 3.$$

3. Build the exponential tower by:

$$\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\vdots}}}$$

That is we define a sequence  $\langle a_n \rangle$  by  $a_1 = \sqrt{2}$  and  $a_{n+1} = \sqrt{2}^{a_n}$  for  $n \geq 1$ . Is the sequence convergent?

4. Guess a closed expression for

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2 + \sqrt{2}}}}}$$

where there are  $n$  nested radicals. Prove your guess by induction.

5. Let  $\alpha$  be a real number such that  $\alpha + \frac{1}{\alpha}$  is an integer. Prove that  $\alpha^n + \frac{1}{\alpha^n}$  is an integer for every  $n \in \mathbb{N}$ .

6. Prove that

$$(n+1)(n+2)\cdots(2n) = 2^n \cdot 1 \cdot 3 \cdot 5 \cdots (2n-1),$$

for any  $n \in \mathbb{N}$ .

7. Prove that all numbers of the form 1007, 10017, 100117, 1001117 and so on are divisible by 53. (1 followed by two zeros and number of 1 increases in each step and the last digit is 7.)